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Ward Identity for Gauge Field in Stochastic Quantization with Nonlocal Form Factors

M. Dineykhan and Kh. Namsrai[1]

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In the framework of the stochastic quantization method with nonlocal form factors, two-, three- and four-point correlation functions for the Yang-Mills field and β function are calculated. A Schwinger-Dyson renormalization program is formulated for regularized QCD_4. It is shown that the gauge fixing due to Zwanziger does not break gauge invariance and that the Ward identity is also fulfilled.

1. INTRODUCTION

In recent years, interest has significantly increased in the study of stochastic quantization (Parisi and Wu, 1981) due to the fact that it has been possible to apply an idea of nonequilibrium statistical mechanics to different theoretical field models (e.g., Migdal, 1986). It turns out that this alternative method of quantization of physical systems (besides the usual formalisms of the canonical quantization and the integral over paths) leads to new ideas and methods in quantum field theory. As mentioned by Bern *et al.* (1987), these achievements include Zwanziger's gauge fixing (Zwanziger, 1981), stochastic stabilization (Greensite and Halpern, 1984) and regularization (Bern *et al.*, 1987; Niemi and Wijewardhana, 1982; Breit *et al.*, 1984; Namiki and Yamanaka, 1984; Bern, 1985), the QCD_4 maps which run in ordinary time (Glaudson and Halpern, 1985; Bern and Chan, 1986), and also numerical applications of the Langevin equation in lattice gauge theory (Hamber and Heller, 1984; Batrouni *et al.*, 1985).

In a previous paper (Dineykhan and Namsrai, 1988) we studied the problem of regularization of stochastic equations of the Langevin and Schwinger-Dyson type within nonlocal quantum field theory (Efimov, 1977,

[1]Institute of Physics and Technology, Academy of Sciences of Mongolian People's Republic, Ulan-Bator, Mongolia.

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1985). The basic idea of our approach was as follows: Instead of the usual equations of the stochastic scheme, the modified versions with nonlocal white noise

$$\eta(x, t) \Rightarrow \Lambda(x, t) = \int (dy)\, K(x-y)\eta(y, t) \tag{1}$$

were considered, where $K(x) = K(\square)\delta^4(x)$ is the nonlocal generalized function constructed by Efimov (1977); t is the fictitious parameter, usually called the "fifth time." Here $\eta(x, t)$ is the local white noise satisfying the condition

$$\langle \eta(x, t)\eta(x', t')\rangle_\eta = 2\delta^4(x-x')\delta(t-t')$$

It turns out that the nonlocal white noise (1) plays a double role in the scheme of the stochastic quantization method: it controls the quantum behavior of the physical systems and at the same time it makes the theory finite in each order of perturbation.

This method has been employed in the study of scalar and gauge fields and also in scalar electrodynamics. This is the second in a series of papers studying covariant-nonlocal regularization of continuum quantum field theory and is devoted to the renormalization problem and to the calculation of the β function for four-dimensional QCD. Here we use the method expounded by Bern *et al.* (1987), where it was used for continuum regularization with meromorphic functions.

In Section 2 the renormalization of the Schwinger-Dyson equation in four-dimensional QCD is studied. The next section is devoted to the calculation of two-, three-, and four-point diagrams and to the definition of the renormalization constants. Our results show that in the given scheme the Ward identity is fulfilled.

2. THE RENORMALIZATION OF THE SCHWINGER-DYSON EQUATION

The basic equations of the stochastic quantization method are the Langevin and Schwinger-Dyson equations. These equations define the behavior of the field function and their interactions and also the dependence on white noise. The field function, interaction constant, and parameter of gauge fixation in these equations are bare. The bare constant of interaction g_0 and fixed gauge parameter α_0 and also the field function A^α_μ are usually expressed through the physical constants $(g, \alpha, A^\alpha_{R\mu})$:

$$\begin{aligned} g_o &= Z_g Z_A^{-3/2} g \\ A^a_\mu(x) &= Z_A^{1/2} A^a_{R\mu}(x) \\ \alpha_0 &= Z_\alpha^{-1} a \end{aligned} \tag{2}$$

where the constants Z_g, Z_A, and Z_α correspond to the renormalization of the interaction constant, wave function, and gauge fixation, respectively. The Langevin equation for the gauge field is written as

$$\frac{\partial A_R^a(x,t)}{\partial t} = -\frac{\delta S}{\delta A_{R\mu}^a} + D_\mu^{ab} Z^b(x,t) + \int (dy)\, K_{xy}^{ab}(\Delta)\eta_\mu^b(y,t) \tag{3}$$

Let us consider each term in (3) separately. Thus, the first term in (3), $\delta S/\delta A$, is expressed by the action S, which is defined in the form

$$S = \int dx\, \mathscr{L}(x)$$

where $\mathscr{L}$ is the Lagrangian of the gauge field

$$\mathscr{L} = -\tfrac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a \tag{4}$$

Here $F_{\mu\nu}^a$ is the stress tensor of the Yang-Mills field:

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_0 f^{abc} A_\mu^b A_\nu^c \tag{5}$$

Then, in accordance with (2) and (5), we get from (3)

$$\mathscr{L}(x) = \mathscr{L}_R(x) + \mathscr{L}_{CT}(x)$$

where $\mathscr{L}_R$ is the renormalized Lagrangian:

$$\mathscr{L}_R = \tfrac{1}{2} A_{R\mu}^a \Box T_{\mu\nu} A_{R\nu}^a + g f^{abc} A_{R\mu}^b (\partial_\mu A_{R\nu}^a) A_{R\nu}^c + \tfrac{1}{4} g^2 f^{abc} f^{anm} A_{R\mu}^b A_{R\nu}^c A_{R\mu}^n A_{R\nu}^m \tag{6}$$

and $\mathscr{L}_{CT}$ is the corresponding counterterm written in the standard form:

$$\mathscr{L}_{CT} = -\tfrac{1}{2}(Z_A - 1) A_{R\mu}^a \Box T_{\mu\nu} A_{R\nu}^a + (Z_g - 1) g f^{abc} A_{R\mu}^b (\partial_\mu A_{R\nu}^a) A_{R\nu}^c + \frac{1}{4}\left(\frac{Z_g^2}{Z_A} - 1\right) g^2 f^{abc} f^{anm} A_{R\mu}^b A_{R\nu}^c A_{R\mu}^n A_{R\nu}^m \tag{7}$$

In accordance with this, the action is also made up of two pasts: $S = S_R + S_{CT}$. Here we have used the notation employed in our previous paper (Dineykhan and Namsrai, 1988). The second expression in (3) is the so-called Zwanziger term (Zwanziger, 1981) and defines the gauge fixation. Due to (2) and after some simplification one gets

$$D_\mu^{ab} Z^b = [\delta^{ab}\partial_\mu + g_0 f^{abc} A_\mu^c(x)] Z^b = Z_A^{-1/2}(Z_R^a + Z_{CT}^a)$$

where

$$Z_R^a = \frac{1}{\alpha}(\delta^{ab}\partial_\mu + g f^{abc} A_{R\mu}^c)\, \partial_\nu A_{R\nu}^b \tag{8}$$

and

$$Z^a_{CT} = \frac{1}{\alpha}\,[(Z_\alpha Z_A - 1)\delta^{ab}\partial_\mu + g(Z_g Z_\alpha - 1) f^{abc} A^C_{R\mu}]\,\partial_\nu A^b_{R\nu} \tag{9}$$

In our case, the distribution of the white noise is the entire analytic function and is caused by the quantum behavior of the system; it consists of two parts:

$$K^{ab}_{xy}(\Delta) = K^{ab}_{xy}(\Delta_R) + K^{ab}_{xy}(\Delta_{CT})$$

where the renormalized part takes the form (see Dineykhan and Namsrai, 1988)

$$\begin{aligned} K^{ab}_{xy}(\Delta_R) = K^{ab}_{xy}(\Box) &+ \frac{g}{2}[K^{(1)}(\Box)\Gamma^R_1 H(\Box) + H(\Box)\Gamma^R_1 K^{(1)}(\Box)]^{ab}_{xy} \\ &+ \frac{g^2}{2}[K^{(1)}(\Box)\Gamma^R_2 H(\Box) + H(\Box)\Gamma^R_2 K^{(1)}(\Box)]^{ab}_{xy} \\ &+ \frac{g^2}{6}[K^{(2)}(\Box)\Gamma^R_1 H(\Box)\Gamma^R_1 H(\Box) \\ &+ H(\Box)\Gamma^R_1 K^{(2)}(\Box)\Gamma^R_1 H(\Box) \\ &+ H(\Box)\Gamma^R_1 H(\Box)\Gamma^R_1 K^{(2)}(\Box)]^{ab}_{xy} \end{aligned} \tag{10}$$

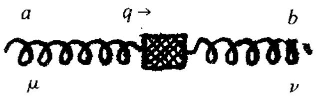
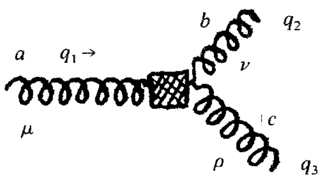
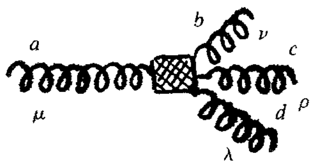
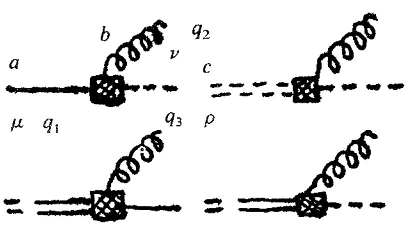
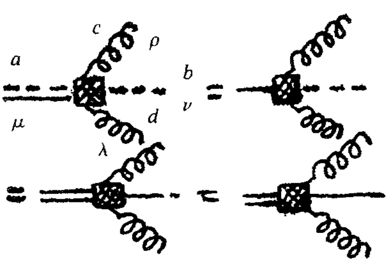
and corresponding counterterm reads

$$\begin{aligned} K^{ab}_{xy}(\Delta_{CT}) = \tfrac{1}{2} g \left(\frac{Z_g}{Z_A} - 1\right) &[K^{(1)}(\Box)\Gamma^R_1 H(\Box) + H(\Box)\Gamma^R_1 K^{(1)}(\Box)]^{ab}_{xy} \\ + \frac{g^2}{2}\left(\frac{Z_g^2}{Z_A^2} - 1\right) &[K^{(1)}(\Box)\Gamma^R_2 H(\Box) + H(\Box)\Gamma^R_2 K^{(1)}(\Box)]^{ab}_{xy} \\ + \frac{g^2}{6}\left(\frac{Z_g^2}{Z_A^2} - 1\right) &[K^{(2)}(\Box)\Gamma^R_1 H(\Box)\Gamma^R_1 H(\Box) \\ + H(\Box)\Gamma^R_1 K^{(2)}(\Box)\Gamma^R_1 H(\Box) &+ H(\Box)\Gamma^R_1 H(\Box)\Gamma^R_1 K^{(2)}(\Box)]^{ab}_{xy} \end{aligned} \tag{11}$$

where $K^{(j)}(\Box)$ is an entire analytic function of the concrete type as shown in (Efimov, 1977); the renormalization vertexes Γ^R_1 and Γ^R_2 are defined in the following way:

$$\begin{aligned} (\Gamma^R_1)^{ab}_{xy} &= l^2 f^{abc}[A^c_{R\mu}(x)\partial_\mu + \partial_\mu A^c_{R\mu}(x)]\ \delta^{(4)}_{(x-y)} \\ (\Gamma^R_2)^{ab}_{xy} &= l^2 f^{amn} f^{ncb} A^m_{R\mu}(x) A^c_{R\mu}(x)\ \delta^{(4)}(x-y) \end{aligned} \tag{12}$$

Table I

Diagram type	Expression
	$\delta^{ab}[(1-Z_A)q^2\delta_{\mu\nu}+(1/\alpha-1+Z_A)$ $\times(1-Z_\alpha/\alpha)q_\mu q_\nu]$
	$\frac{1}{2}[(Z_g-1)T^{abc}_{\mu\nu\rho}$ $+1/\alpha(Z_\alpha Z_g-1)Z^{abc}_{\mu\nu\rho}]$
	$\frac{1}{6}(Z_g^2/Z_A-1)\,Q^{abcd}_{\mu\nu\rho\eta}$
	$ig(Z_g/Z_A-1)f^{abc}l^2(q_1-q_3)_\nu$
	$g^2(Z_g^2/Z_A^2-1)f^{and}f^{nbc}$ $\times\, l^2\delta_{\lambda\rho}\delta_{\mu\nu}$

Making the necessary calculations, taking account of (6), (8), and (10), we obtain the renormalized Schwinger-Dyson equations in the x space:

$$\left\langle\int d^4x\left[-\frac{\delta S_R}{\delta A^a_{R\mu}}+Z^a_{R\mu}+\int (dy)\,(dz)K^{bc}_{yz}(\Delta_R)\right.\right.$$
$$\left.\left.\times\frac{\delta}{\delta A^c_{R\mu}}\,K^{ba}_{xy}(\Delta_R)\right]\frac{\delta F[A_R]}{\delta A^a_{R\mu}}\right\rangle=0 \qquad (13)$$

Using equation (13), one should calculate the correlation function which determine two-, three-, and four-point renormalization. On the other hand, to determine the renormalization constants Z_g, Z_A, and Z_α, we must calculate the corresponding counterterm diagrams. In accordance with (7), (9), and (11) it is simplest to calculate the vertex diagrams and these expressions are shown in Table I. Here we use the following tensor notation:

$$\begin{aligned}
T^{abc}_{\mu\nu\rho} &\equiv T^{abc}_{\mu\nu\rho}(q_1, q_2, q_3)\\
&= -igf^{abc}[(q_1-q_2)_\rho\delta_{\mu\nu}+(q_2-q_3)_\mu\delta_{\nu\rho}+(q_3-q_1)_\nu\delta_{\mu\rho}]\\
Z^{abc}_{\mu\nu\rho} &\equiv Z^{abc}_{\mu\nu\rho}(q_1, q_2, q_3)\\
&= -igf^{abc}[q_{3\rho}\delta_{\mu\nu}-q_{2\nu}\delta_{\mu\rho}] \qquad (14)\\
Q^{abcd}_{\mu\nu\rho\eta} &= -g^2[f^{abn}f^{cdn}(\delta_{\mu\rho}\delta_{\nu\eta}-\delta_{\mu\eta}\delta_{\nu\rho})\\
&\quad +f^{acn}f^{bdn}(\delta_{\mu\nu}\delta_{\rho\eta}-\delta_{\mu\eta}\delta_{\nu\rho})+f^{adn}f^{cbn}(\delta_{\mu\rho}\delta_{\nu\eta}-\delta_{\mu\nu}\delta_{\rho\eta})]
\end{aligned}$$

3. TWO-POINT RENORMALIZATION

Let us consider two-point renormalization for the Yang-Mills field. From (13) one can determine for the Schwinger-Dyson equations the correlation $\langle A^a_{R\mu}(q_1)A^b_{R\nu}(q_2)\rangle$. In Dineykhan, Namsrai (1988) we proposed a method to calculate the correlation function in the framework of the stochastic quantization method with nonlocal form factors. Details of the concrete calculations are not given here. Two-point renormalization diagrams are presented in Figure 1. The contributions corresponding to the diagram on Figure 1a are written in the following way:

$$\langle A^a_{R\mu}(q_1)A^b_{R\nu}(q_2)\rangle=\delta^d(q_1+q_2)\Pi^{ab}_{R\mu\nu}(q_1)$$

where

$$\begin{aligned}
\Pi^{ab}_{R\mu\nu}(q)=\frac{2}{q^2}\int (dp)\frac{V(p^2l^2)}{p^2}\,\frac{1}{p^2+q^2+(p-q)^2}&\left[\frac{1}{q^2}\Gamma^{amn}_{\mu\rho\beta}(q, p, p-q)\right.\\
\times\Gamma^{nmb}_{\beta\rho\nu}(q-p, p, -q)+\frac{1}{q^2}\Gamma^{amn}_{\mu\rho\beta}&(q, p-q, -p)\Gamma^{mnb}_{\rho\beta\nu}(q-p, p, -q)\\
+\frac{1}{(p-q)^2}\Gamma^{amn}_{\mu\rho\beta}(q-p, p, -q)&\left.\Gamma^{bmn}_{\nu\rho\beta}(-q, p, q-p)\right] \qquad (15)
\end{aligned}$$

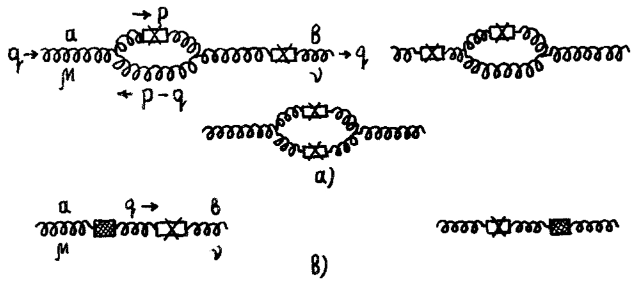


Fig. 1. (a) Two-point renormalization diagrams. (b) Two-point counterterm diagrams.

and the three-gluon vertex $\Gamma^{bmn}_{\nu\rho\beta}$ is defined in the standard way (see Ramond, 1981)

$$\Gamma^{bmn}_{\nu\rho\beta}(q_1, q_2, q_3) = \frac{1}{2}\left(T^{bmn}_{\mu\rho\beta} + \frac{1}{\alpha} Z^{bmn}_{\mu\rho\beta} \right) \tag{16}$$

The $V(p^2 l^2)$ are the nonlocal form factors (Efimov, 1977). Using the Mellin representation

$$V(p^2 l^2) = \frac{1}{2i} \int_{-\beta+i\infty}^{-\beta-i\infty} d\xi \frac{v(\xi)}{\sin \pi\xi} (l^2 p^2)^{\xi}, \qquad 0<\beta<1 \tag{17}$$

and making a simple calculation, from (15) one can obtain easily

$$\Pi^{ab}_{R,\mu\nu}(q) = \frac{g^2 \delta^{ab} N}{16\pi^2} \frac{\ln \mu^2 l^2}{q^2} \left(-\frac{8}{3} \delta_{\nu\mu} + \frac{43}{24} \frac{q_\nu q_\mu}{q^2} \right) \tag{18}$$

where N is of the order of $SU(N)$ group symmetry and μ is an undetermined parameter which corresponds to the choice of renormalization scale. The counterterm contribution is determined by the diagram shown in Figure 1b. After some calculations we have

$$\Pi^{ab}_{CT,\mu\nu}(q) = \frac{\delta^{ab}}{q^2} \left[(1 - Z_A)\delta_{\mu\nu} + Z_A(1 - Z_\alpha) \frac{q_\mu q_\nu}{q^2} \right] \tag{19}$$

Using (18) and (19) and requiring that the R sum plus the CT sum equals zero gives immediately

$$Z_A^{(2)} = 1 - \frac{8}{3} \frac{g^2 N}{16\pi^2} \ln l^2 \mu^2$$

$$Z_\alpha^{(2)} = 1 + \frac{43}{24} \frac{g^2 N}{16\pi^2} \ln l^2 \mu^2 \tag{20}$$

4. THREE-POINT RENORMALIZATION

Let us consider the three-point renormalization in the momentum space. From (13) the Schwinger–Dyson equation gives for the three-point correlation function

$$
\begin{aligned}
&G^{a_1a_2a_3}_{\nu_1\nu_2\nu_3}(q_1, q_2, q_3)\\
&\quad\equiv\langle A^{a_1}_{R\nu_1}(q_1)A^{a_2}_{R\nu_2}(q_2)A^{a_3}_{R\nu_3}(q_3)\rangle\\
&\quad=\frac{1}{q_1^2+q_2^2+q_3^2}\\
&\quad\times\int(dp_1)(dp_2)[\bar{\delta}^4(q_1-p_1-p_2)\\
&\quad\times\Gamma^{a_1bc}_{\nu_1\theta\mu}(-q_1, p_1, p_2)\langle A^c_{R\mu}(p_2)A^b_{R\theta}(p_1)A^{a_2}_{R\nu_2}(q_2)\\
&\quad\times A^{a_3}_{R\nu_3}(q_3)\rangle+\text{cyclic perm in }(q)]\\
&\quad+\frac{1}{q_1^2+q_2^2+q_3^2}\int(dp_1)(dp_2)(dp_3)[\bar{\delta}^4(p_1+p_2+p_3-q_1)\\
&\quad\times\Gamma^{a_1cnm}_{\nu_1\gamma\beta\mu}\langle A^m_{R\mu}(p_3)A^n_{R\beta}(p_2)A^c_{R\gamma}(p_1)A^{a_2}_{R\nu_2}(q_2)A^{a_3}_{R\nu_3}(q_3)\rangle\\
&\quad+\text{cyclic perm in }(q)]
\end{aligned}
\tag{21}
$$

where $\bar{\delta}^4(p)=(2\pi)^4_\delta(p)$ and the four-gluon vertex is defined in the standard way (Ramond, 1981) as

$$\Gamma^{a_1a_2a_3a_4}_{\nu_1\nu_2\nu_3\nu_4}=\tfrac{1}{6}Q^{a_1a_2a_3a_4}_{\nu_1\nu_2\nu_3\nu_4} \tag{22}$$

From (21) it is shown that the three-point correlation functions are expressed through four- and five-point correlation functions. If we restricted ourselves to only the first order of g, then from (21) we have

$$
\begin{aligned}
&G^{(1)a_1a_2a_3}_{\nu_1\nu_2\nu_3}(q_1, q_2, q_3)\\
&\quad=\frac{2\bar{\delta}^4(q_1+q_2+q_3)}{q_1^2+q_2^2+q_3^2}\left[\frac{1}{q_2^2q_3^2}\Gamma^{a_1a_2a_3}_{\nu_1\nu_2\nu_3}(q_1, q_2, q_3)\right.\\
&\quad\left.+\frac{1}{q_1^2q_3^2}\Gamma^{a_2a_3a_1}_{\nu_2\nu_3\nu_1}(q_2, q_3, q_1)+\frac{1}{q_1^2q_2^2}\Gamma^{a_3a_1a_2}_{\nu_3\nu_1\nu_2}(q_3, q_1, q_2)\right]
\end{aligned}
$$

This expression determines the general structure of the three-point renormalization, and corresponding diagrams are shown in Figure 2. Three-point renormalization is determined by the diagrams represented in Figures 3–7. Four- and five-point correlation functions entering in (21) may be expressed by six-point correlation functions. Using (21), one can obtain the expressions corresponding to the diagrams shown in Figures 3–5. In particular, the

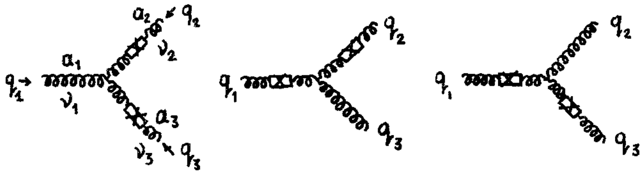


Fig. 2. Three-point structural diagrams.

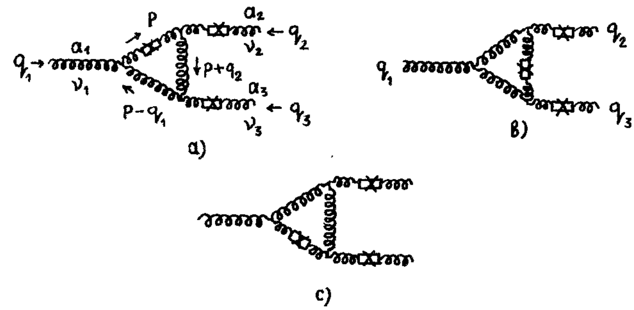


Fig. 3. Three-point renormalization vertex diagrams.

expression corresponding to the diagram shown in Figure 3a is written in the following form:

$$
\begin{aligned}
&G^{(3a)a_1a_2a_3}_{\nu_1\nu_2\nu_3}(q_1, q_2, q_3)\\
&\quad=\frac{8}{q_1^2+q_2^2+q_3^2}\,\frac{1}{q_2^2q_3^2}\int (dp)\,\frac{V(p^2l^2)}{p^2}\\
&\qquad\times\Gamma^{a_1b_1b_2}_{\nu_1\mu_1\mu_2}(q_1, -p_1, p-q_1)\,\frac{1}{p^2+(p-q_1)^2+q_2^2+q_3^2}\\
&\qquad\times\Gamma^{b_2c_1a_3}_{\mu_2\beta_1\nu_3}(q_1-p, q_2+p, q_3)\,\frac{1}{p^2+(p+q_2)^2+q_2^2+2q_3^2}\\
&\qquad\times\Gamma^{c_1a_2b_1}_{\beta_1\nu_2\mu_1}(-p-q_2, q_2, p)
\end{aligned}
\qquad (23)
$$

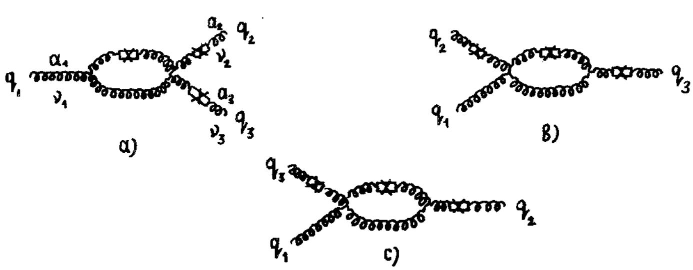


Fig. 4. Three-point renormalization vertex diagrams.

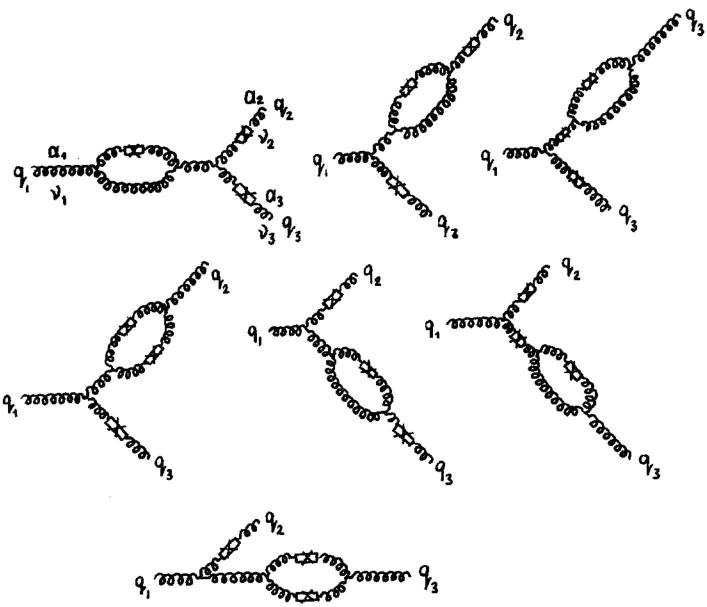


Fig. 5. Three-point renormalization tree-structure diagrams.

Using the representation (17) and according to (16) and after some simplification and integration over (dp), we have

$$G^{(3a)a_1a_2a_3}_{\nu_1\nu_2\nu_3}(q_2, q_2, q_3)$$
$$= \frac{ig^3 f^{a_1a_2a_3} N}{16\pi^2 q_2^2 q_3^2} \frac{\ln l^2\mu^2}{q_1^2 + q_2^2 - q_3^2}$$
$$\times \left[\tfrac{7}{24} q_{2\nu_2}\delta_{\nu_1\nu_3} - \tfrac{49}{48} q_{2\nu_3}\delta_{\nu_1\nu_2} + \tfrac{31}{96} q_{2\nu_1}\delta_{\nu_2\nu_3}\right.$$
$$\left. + \tfrac{5}{24} q_{3\nu_2}\delta_{\nu_1\nu_3} + \tfrac{5}{24} q_{3\nu_3}\delta_{\nu_1\nu_2} - \tfrac{37}{96} q_{3\nu_1}\delta_{\nu_2\nu_3}\right]$$

The expressions for the diagrams of Figures 3b and 3c may be written in an analogous form as (23) and we obtain the contribution from the diagrams

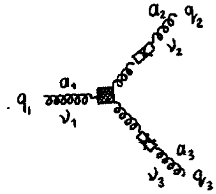


Fig. 6. Three-point counterterm vertex diagram.

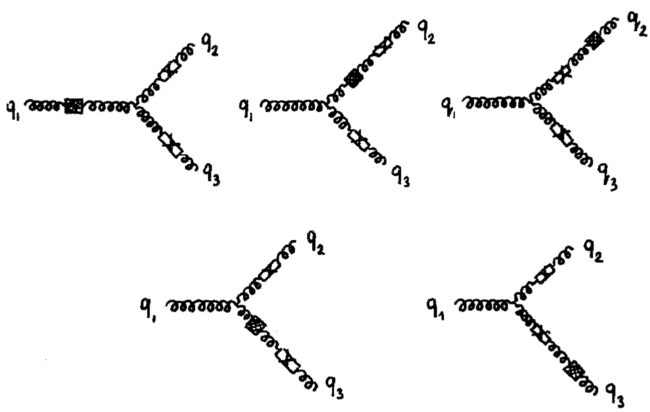


Fig. 7. Three-point counterterm tree-structure diagrams.

of Figures 3a-3c:

$$G^{(3)a_1a_2a_3}_{\nu_1\nu_2\nu_3}(q_1, q_2, q_3)$$
$$=\frac{ig^3 f_{a_1a_2a_3}N}{16\pi^2 q_2^2 q_3^2}\frac{\ln l^2\mu^2}{q_1^2+q_2^2+q_3^2}$$
$$\times\left(-\tfrac{85}{48}q_{2\nu_1}\delta_{\nu_2\nu_3}+\tfrac{7}{24}q_{2\nu_2}\delta_{\nu_1\nu_3}+\tfrac{125}{48}q_{2\nu_3}\delta_{\nu_1\nu_2}\right.$$
$$\left.-\tfrac{125}{48}q_{3\nu_2}\delta_{\nu_1\nu_3}-\tfrac{7}{24}q_{3\nu_3}\delta_{\nu_1\nu_2}+\tfrac{85}{48}q_{3\nu_1}\delta_{\nu_2\nu_3}\right) \quad (24)$$

The expression for the diagram illustrated in Figure 4a, is written in the following way:

$$G^{(4a)a_1a_2a_3}_{\nu_1\nu_2\nu_3}(q_1, q_2, q_3)$$
$$=\frac{12}{q_1^2+q_2^2+q_3^2}\frac{1}{q_2^2q_3^2}\int(dp)\frac{V(p^2l^2)}{p^2}\Gamma^{a_1cb}_{\nu_1\theta\beta}(q_1-p, p-q_1)$$
$$\times\frac{1}{p^2+(p-q_1)^2+q_2^2+q_3^2}\Gamma^{bca_3a_2}_{\beta\theta\nu_3\nu_2}$$

According to (16), (17), and (22) and making an integration over (dp) we have

$$G^{(4a)a_1a_2a_3}_{\nu_1\nu_2\nu_3}(q_1, q_2, q_3)=\frac{ig^3Nf^{a_1a_2a_3}}{16\pi^2q_2^2q_3^2}\frac{\ln l^2\mu^2}{q_1^2+q_2^2q_3^2}$$
$$\times\left[\tfrac{3}{2}(q_2+q_3)_{\nu_3}\delta_{\nu_1\nu_2}-\tfrac{3}{2}(q_2+q_3)_{\nu_2}\delta_{\nu_1\nu_3}\right] \quad (25)$$

The diagrams of Figures 4b and 4c are calculated in an analogous way as above and we obtain for the diagrams of Figure 4

$$G^{(4)a_1a_2a_3}_{\nu_1\nu_2\nu_3}(q_1, q_2, q_3) = \frac{ig^3 N f^{a_1a_2a_3}}{16\pi^2 q_2^2 q_3^2} \frac{\ln l^2\mu^2}{q_1^2+q_2^2+q_3^2} \times \left(\tfrac{39}{16} q_{2\nu_1}\delta_{\nu_2\nu_3} + \tfrac{3}{2} q_{2\nu_2}\delta_{\nu_1\nu_2} + \tfrac{63}{16} q_{2\nu_3}\delta_{\nu_1\nu_2} + \tfrac{63}{16} q_{3\nu_2}\delta_{\nu_1\nu_3} - \tfrac{3}{2} q_{3\nu_3}\delta_{\nu_1\nu_3} - \tfrac{39}{16} q_{3\nu_1}\delta_{\nu_2\nu_3}\right) \quad (26)$$

Using the notation in (14) and the summation of (26) and (24), we obtain the contributions for the vertex diagram:

$$G^{(\mathrm{ver})a_1a_2a_3}_{R,\nu_1\nu_2\nu_3}(q_1, q_2, q_3) = \frac{g^2 N \ln l^2\nu^2}{16\pi^2 q_2^2 q_3^2} \frac{1}{q_1^2+q_2^2+q_3^2}\left[\tfrac{2}{3}\left(T^{a_1a_2a_3}_{\nu_1\nu_2\nu_3} + Z^{a_1a_2a_3}_{\nu_1\nu_2\nu_3}\right) - \tfrac{43}{24} Z^{a_1a_2a_3}_{\nu_1\nu_2\nu_3}\right] \quad (27)$$

The corresponding counterterm diagrams for this vertex are represented in Figure 6 and their contributions are written in the following way:

$$G^{(\mathrm{ver})a_1a_2a_3}_{CT,\nu_1\nu_2\nu_3}(q_1, q_2, q_3) = \frac{1}{q_1^2+q_2^2+q_3^2} \frac{1}{q_2^2 q_3^2}\left[(Z_g - 1) T^{a_1a_2a_3}_{\nu_1\nu_2\nu_3} + (Z_\alpha Z_g - 1) Z^{a_1a_2a_3}_{\nu_1\nu_2\nu_3}\right] \quad (28)$$

The tree-structural diagrams represented in Figure 7 also contribute to the three-point correlation function. These diagrams are calculated in the same way as above and finally we have the result

$$G^{(\mathrm{tr})a_1a_2a_3}_{R,\nu_1\nu_2\nu_3}(q_1, q_2, q_3) = \frac{g^2 N \ln l^2\mu^2}{16\pi^2 q_2^2 q_3^2} \frac{1}{q_1^2+q_2^2+q_3^2}\Bigg\{-\tfrac{13}{2}\left(T^{a_1a_2a_3}_{\nu_1\nu_2\nu_3} + Z^{a_1a_2a_3}_{\nu_1\nu_2\nu_3}\right) + \frac{43}{24(q_1^2+q_2^2+q_3^2)}\left[q_{1\nu_1}q_{1\sigma_1}\left(T^{a_1a_2a_3}_{\sigma_1\nu_2\nu_3} + Z^{a_1a_2a_3}_{\sigma_1\nu_2\nu_3}\right) + q_{2\nu_2}q_{2\sigma_2} \times \left(T^{a_1a_2a_3}_{\nu_1\sigma_2\nu_3} + Z^{a_1a_2a_3}_{\nu_1\sigma_2\nu_3}\right) + q_{3\nu_3}q_{3\sigma_3}\left(T^{a_1a_2a_3}_{\nu_1\nu_2\sigma_3} + Z^{a_1a_2a_3}_{\nu_1\nu_2\sigma_3}\right)\right] + \frac{43}{24}\left[\frac{q_{2\nu_2}q_{2\sigma_2}}{q_2^2} \times \left(T^{a_1a_2a_3}_{\nu_1\sigma_2\nu_3} + Z^{a_1a_2a_3}_{\nu_1\sigma_2 a_3}\right) + \frac{q_{3\nu_3}q_{3\sigma_3}}{q_3^2}\left(T^{a_1a_2a_3}_{\nu_1\nu_2\sigma_3} + Z^{a_1a_2a_3}_{\nu_1\nu_2\sigma_3}\right)\right]\Bigg\} \quad (29)$$

The corresponding counterterms for these diagrams are represented in

Figure 8 and their contribution is given by

$$G^{(\mathrm{tr})a_1a_2a_3}_{CT,\nu_1\nu_2\nu_3}(q_1, q_2, q_3)$$

$$= \frac{1}{q_1^2+q_2^2+q_3^2}\frac{1}{q_2^2q_3^2}\left\{3(1-Z_A)[T^{a_1a_2a_3}_{\nu_1\nu_2\nu_3}+Z^{a_1a_2a_3}_{\nu_1\nu_2\nu_3}]\right.$$

$$-\frac{Z_A(Z_\alpha-1)}{q_1^2+q_2^2+q_3^2}[q_{1\nu_1}q_{1\sigma_1}(T^{a_1a_2a_3}_{\sigma_1\nu_2\nu_3}+Z^{a_1a_2a_3}_{\sigma_1\nu_2\nu_3})+q_{2\nu_2}q_{2\sigma_2}(T^{a_1a_2a_3}_{\nu_1\sigma_2\nu_3}$$

$$+Z^{a_1a_2a_3}_{\nu_1\sigma_2\nu_3})+q_{3\sigma_3}q_{3\nu_3}(T^{a_1a_2a_3}_{\nu_1\nu_2\sigma_3}+Z^{a_1a_2a_3}_{\nu_1\nu_2\sigma_3})]-Z_A(Z_\alpha-1)$$

$$\left.\times\left[\frac{q_{2\nu_2}q_{2\sigma_2}}{q_2^2}(T^{a_1a_2a_3}_{\nu_1\sigma_2\nu_3}+Z^{a_1a_2a_3}_{\nu_1\sigma_2\nu_3})+\frac{q_{3\nu_3}q_{3\sigma_3}}{q_3^2}(T^{a_1a_2a_3}_{\nu_1\nu_2\sigma_3}+Z^{a_1a_2a_3}_{\nu_1\nu_2\sigma_3})\right]\right\} \tag{30}$$

Adding (27)-(30), i.e., $G^{\mathrm{ver}}_R+G^{\mathrm{ver}}_{CT}+G^{\mathrm{tr}}_R+G^{\mathrm{tr}}_{CT}=0$, inserting the value of Z_A defined in (20) into it, and equating to zero every tensor coefficients separately, we have

$$Z^{(3)}_\alpha = 1+\frac{43}{24}\frac{Ng^2}{16\pi^2}\ln l^2\mu^2$$

$$Z^{(3)}_g = 1-\frac{13}{6}\frac{g^2N}{16\pi^2}\ln l^2\mu^2 \tag{31}$$

From (20) and (31) we see that Z_α and Z_A satisfy the Ward identity:

$$Z^{(2)}_\alpha = Z^{(3)}_\alpha;\qquad Z^{(2)}_A = Z^{(3)}_A$$

Now let us consider verification of the Ward identity for Z_g. For this purpose, we consider four-point renormalization.

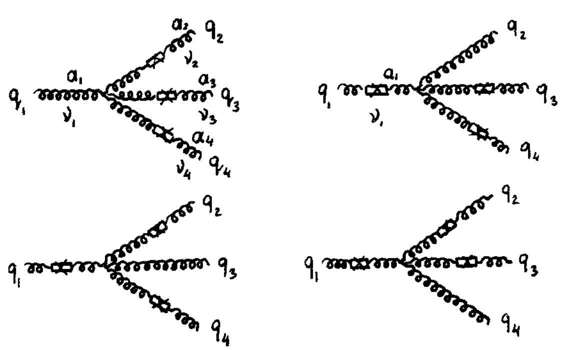


Fig. 8. Four-point structural diagrams.

5. FOUR-POINT RENORMALIZATION

Four-point renormalization is defined by the set of diagrams shown in Figures 9–12. If it is limited to the lower order of g, then the four-point correlation function is defined by

$$G^{(1)a_1a_2a_3a_4}_{\nu_1\nu_2\nu_3\nu_4}(q_1, q_2, q_3, q_4) = \frac{\bar{\delta}^4(q_1+q_2+q_3+q_4)}{q_1^2+q_2^2+q_3^2+q_4^2}\left[\frac{1}{q_2^2q_3^2q_4^2} Q^{a_1a_2a_3a_4}_{\nu_1\nu_2\nu_3\nu_4} + \frac{1}{q_1^2q_3^2q_4^2} Q^{a_2a_3a_4a_1}_{\nu_1\nu_2\nu_3\nu_4} + \frac{1}{q_1^2q_2^2q_4^2} Q^{a_3a_4a_1a_2}_{\nu_3\nu_4\nu_1\nu_2} + \frac{1}{q_1^2q_2^2q_3^2} Q^{a_4a_1a_2a_3}_{\nu_4\nu_1\nu_2\nu_3}\right]$$

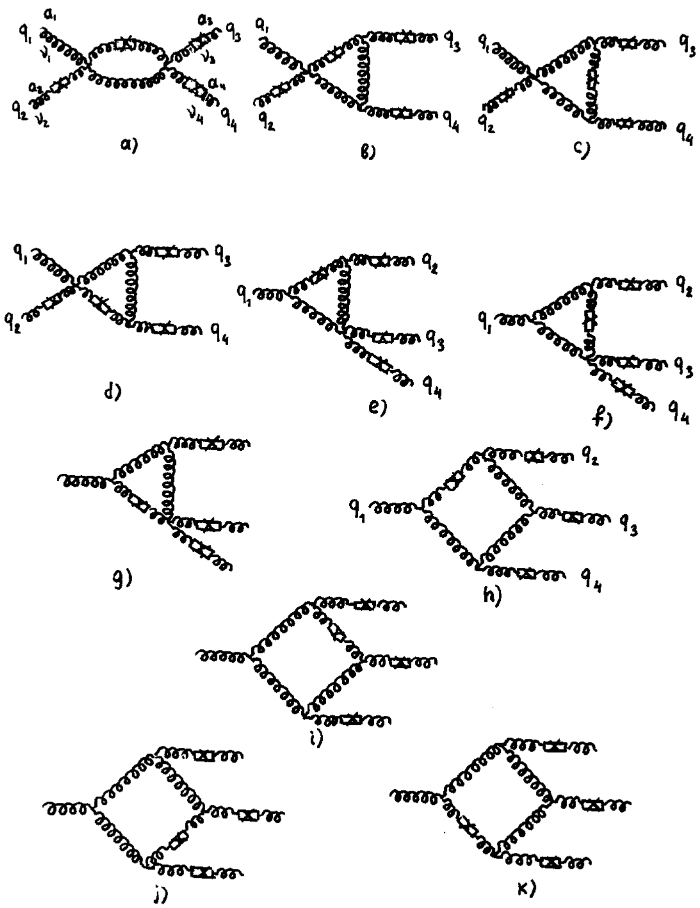


Fig. 9. Four-point renormalization vertex diagrams. Cyclic permutations for q_2, q_3, q_4.

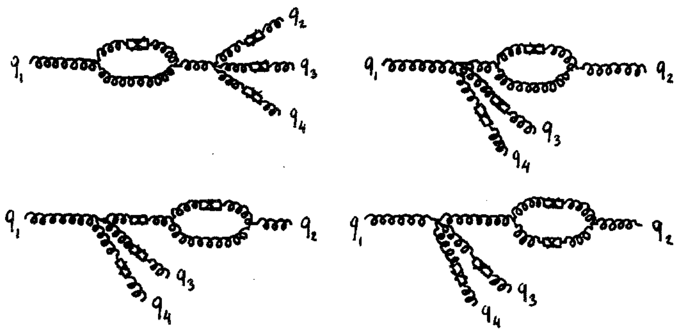


Fig. 10. Four-point renormalization tree-structure diagrams. Cyclic permutations for q_2, q_3, q_4.

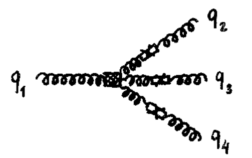


Fig. 11. Four-point counterterm diagram.

The corresponding diagrams are represented in Figure 8. This expression defines the common structure of the four-point renormalization. The expression which defines the four-point correlation function is obtained from the Schwinger-Dyson equation (13). The full expression is complicated and therefore we do not write it explicitly as was done in the case of the three-point correlation function (21). However, we give concrete expressions corresponding to separate diagrams. We consider only expressions corresponding to the diagram shown in Figure 8a, and others will be obtained

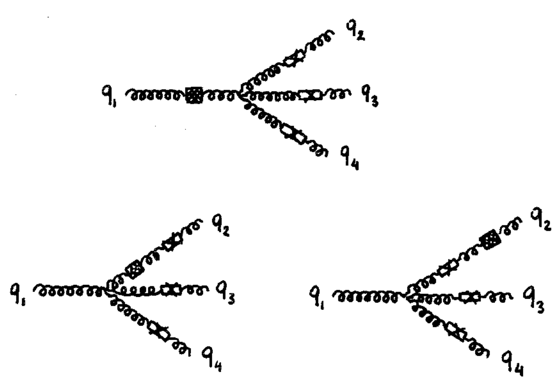


Fig. 12. Four-point counterterm tree-structure diagrams. Cyclic permutations for q_2, q_3, q_4.

by cyclic permutation. First we consider the vertex diagram. The expression corresponding to the diagram of Figure 9a is

$$G^{(9a)a_1a_2a_3a_4}_{\nu_1\nu_2\nu_3\nu_4} = \frac{36}{q_2^2q_3^2q_4^2}\frac{1}{q_1^2+q_2^2+q_3^2+q_4^2}\int (dp)\frac{V(p^2l^2)}{p^2}\Gamma^{a_1b_1b_2a_2}_{\nu_1\mu_1\mu_2\nu_2} \times \frac{1}{(p+q_3+q_4)^2+p^2+q_3^2+q_4^2+2q_2^2}\Gamma^{b_1a_4a_3b_2}_{\mu_1\nu_4\nu_3\mu_2}$$

where the Γ-four-point vertex constant is defined in (22). Using the notation (17) and after some algebraic simplifications, we have

$$\begin{aligned} G^{(9a)a_1a_2a_3a_4}_{\nu_1\nu_2\nu_3\nu_4} &= \frac{-g^4}{q_1^2+q_2^2+q_3^2+q_4^2}\frac{\ln l^2\mu^2}{16\pi^2q_2^2q_3^2q_4^2}[2Nf_{a_1a_2n}f_{a_3a_4n} \\ &\times(\delta_{\nu_1\nu_3}\delta_{\nu_2\nu_4}-\delta_{\nu_1\nu_4}\delta_{\nu_2\nu_3})+S^{a_1a_2a_3a_4}(2\delta_{\nu_1\nu_2}\delta_{\nu_3\nu_4}+\delta_{\nu_1\nu_4}\delta_{\nu_2\nu_3}) \\ &+S^{a_1a_2a_4a_3}(2\delta_{\nu_1\nu_2}\delta_{\nu_3\nu_4}+\delta_{\nu_2\nu_4}\delta_{\nu_1\nu_3})] \end{aligned} \tag{32}$$

Here we have introduced the following notation:

$$S^{a_1a_2a_3a_4}=f^{a_1nm}f^{a_2mc}f^{a_3cb}f^{a_4bn} \tag{33}$$

Let us consider the diagram of Figure 9b. The corresponding expression has the form

$$\begin{aligned} G^{(9b)a_1a_2a_3a_4}_{\nu_1\nu_2\nu_3\nu_4} &= \frac{24}{q_1^2+q_2^2+q_3^2+q_4^2}\frac{1}{q_2^2q_3^2q_4^2}\Gamma^{a_1b_1b_2a_2}_{\nu_1\mu_1\mu_2\nu_2}\int (dp)\frac{V(p^2l^2)}{p^2} \\ &\times\Gamma^{b_1c_1a_4}_{\mu_1\beta_1\nu_4}(-q_3-q_4-p, q_3+p, q_4) \\ &\times\frac{1}{(p+q_3+q_4)^2+p^2+q_3^2+q_4^2+2q_2^2}\Gamma^{c_1a_3b_2}_{\beta_1\nu_3\mu_2}(-q_3-p, q_3, p) \\ &\times[(p+q_3)^2+p^2+q_3^2+2q_2^2+2q_4^2]^{-1} \end{aligned}$$

The three-gluon vertex $\Gamma^{b_1b_2b_3}_{\mu_1\mu_2\mu_3}$ is obtained in (16). After some standard calculations we have

$$\begin{aligned} G^{(9b)a_1a_2a_3a_4}_{\nu_1\nu_2\nu_3\nu_4} &= \frac{-g^4}{q_1^2+q_2^2+q_3^2+q_4^2}\frac{\ln l^2\mu^2}{16\pi^2q_2^2q_3^2q_4^2}\Big[\frac{N}{32}f^{a_1a_2n}f^{a_3a_4n}(\delta_{\nu_1\nu_3}\delta_{\nu_2\nu_4}-\delta_{\nu_2\nu_3}\delta_{\nu_1\nu_4}) \\ &+S^{a_1a_2a_3a_4}(\tfrac{9}{16}\delta_{\nu_1\nu_2}\delta_{\nu_3\nu_4}+\tfrac{1}{8}\delta_{\nu_2\nu_3}\delta_{\nu_1\nu_4}+\tfrac{1}{16}\delta_{\nu_1\nu_3}\delta_{\nu_2\nu_4}) \\ &+S^{a_1a_2a_4a_3}(\tfrac{9}{16}\delta_{\nu_1\nu_2}\delta_{\nu_3\nu_4}+\tfrac{1}{16}\delta_{\nu_2\nu_3}\delta_{\nu_1\nu_4}+\tfrac{1}{8}\delta_{\nu_1\nu_3}\delta_{\nu_2\nu_4})\Big] \end{aligned} \tag{34}$$

Now consider the diagram of Figure 9c; its corresponding expression has the following form

$$G^{(9c)a_1a_2a_3a_4}_{\nu_1\nu_2\nu_3\nu_4}$$

$$=\frac{48}{q_1^2+q_2^2+q_3^2+q_4^2}\frac{1}{q_2^2q_3^2q_4^2}$$

$$\times\Gamma^{a_1b_1b_2a_2}_{\nu_1\mu_1\mu_2\nu_2}\int(dp)\frac{V(p^2l^2)}{p^2}\Gamma^{b_1c_1a_3}_{\mu_1\beta_1\nu_3}(p-q_3,-p,q_3)$$

$$\times\frac{1}{(p+q_4)^2+(p-q_3)^2+q_3^2+q_4^2+2q_2^2}\Gamma^{b_2a_4c_1}_{\mu_2\nu_4\beta_1}(-q_4+p,q_4,p)$$

$$\times[p^2+(p+q_4)^2+q_4^2+2q_2^2+2q_3^2]^{-1}$$

Carrying out analogous calculations to the above, we find

$$G^{(9c)a_1a_2a_3a_4}_{\nu_1\nu_2\nu_3\nu_4}$$

$$=\frac{-2g^4}{q_1^2+q_2^2+q_3^2+q_4^2}\frac{\ln l^2\mu_2}{16\pi^2q_2^2q_3^2q_4^2}$$

$$\times\left[\frac{N}{8}f^{a_1a_2n}f^{a_3a_4n}(\delta_{\nu_1\nu_3}\delta_{\nu_2\nu_4}-\delta_{\nu_1\nu_4}\delta_{\nu_2\nu_3})\right.$$

$$+S^{a_1a_2a_4a_3}(\tfrac{11}{16}\delta_{\nu_1\nu_2}\delta_{\nu_3\nu_4}+\tfrac{1}{4}\delta_{\nu_1\nu_3}\delta_{\nu_2\nu_4})$$

$$\left.+S^{a_1a_2a_3a_4}(\tfrac{11}{16}\delta_{\nu_1\nu_2}\delta_{\nu_2\nu_4}+\tfrac{1}{4}\delta_{\nu_1\nu_4}\delta_{\nu_2\nu_3})\right] \tag{35}$$

From (35) and (34) one can see that the structure of the corresponding expression for the diagrams depends to a great extent on the permutation of internal lines. The diagram of Figure 9d is calculated in an analogous way and it is equal to the expression defined in (34). The corresponding expression for the diagram of Figure 9c is

$$G^{(9e)a_1a_2a_3a_4}_{\nu_1\nu_2\nu_3\nu_4}$$

$$=\frac{24}{q_1^2+q_2^2+q_3^2+q_4^2}\frac{1}{q_2^2q_3^2q_4^2}\int(dp)\frac{V(p^2l^2)}{p^2}\Gamma^{b_1c_1a_4a_3}_{\mu_1\beta_1\nu_4\nu_3}$$

$$\times\frac{1}{p^2+(p-q_1)^2+q_2^2+q_3^2+q_4^2}\Gamma^{a_1b_1b_2}_{\nu_1\mu_1\mu_2}(q_1,p-q_1,-p)$$

$$\times\frac{1}{p^2+(p+q_2)^2+q_2^2+2q_3^2+2q_4^2}\Gamma^{c_1a_2b_2}_{\beta_1\nu_2\mu_2}(-p-q_2,q_2,p)$$

After a simple calculation we obtain

$$G^{(9e)a_1a_2a_3a_4}_{\nu_1\nu_2\nu_3\nu_4} = \frac{-g^4}{q_1^2+q_2^2+q_3^2+q_4^2}\,\frac{\ln l^2\mu^2}{16\pi^2 q_2^2q_3^2q_4^2} \times \left[\frac{N}{8} f^{a_1a_2n}f^{a_3a_4n}(\delta_{\nu_1\nu_3}\delta_{\nu_2\nu_4}-\delta_{\nu_2\nu_3}\delta_{\nu_1\nu_4}) + S^{a_1a_2a_3a_4}(\tfrac{5}{8}\delta_{\nu_1\nu_2}\delta_{\nu_3\nu_4}+\tfrac{3}{8}\delta_{\nu_2\nu_3}\delta_{\nu_1\nu_4}+\tfrac{1}{8}\delta_{\nu_1\nu_3}\delta_{\nu_2\nu_4}) + S^{a_1a_2a_4a_3}(\tfrac{5}{8}\delta_{\nu_1\nu_2}\delta_{\nu_3\nu_4}+\tfrac{1}{8}\delta_{\nu_2\nu_3}\delta_{\nu_1\nu_4}+\tfrac{3}{8}\delta_{\nu_1\nu_3}\delta_{\nu_2\nu_4})\right] \quad (36)$$

Other diagrams are calculated in an analogous way. We do not go into the details of the calculation; the results are given in Table II. Here it is necessary

Table II

Diagram type	Contributions[a]
Renormalization vertex diagram: Figure 9 + (q_2, q_3, q_4) permutations	$-\frac{1}{6}CQ^{a_1a_2a_3a_4}_{\nu_1\nu_2\nu_3\nu_4}$
Renormalization tree-structure diagram: Figure 10+(q_2, q_3, q_4) permutations	$\frac{55C}{6}Q^{a_1a_2a_3a_4}_{\nu_1\nu_2\nu_3\nu_4} - \frac{43C}{24(q_1^2+q_2^2+q_3^2+q_4^2)} \times [q_{1\nu_1}q_{1\sigma_1}Q^{a_1a_2a_3a_4}_{\sigma_1\nu_2\nu_3\nu_4} + q_{2\nu_2}q_{2\sigma_2}Q^{a_1a_2a_3a_4}_{\nu_1\sigma_2\nu_3\nu_4} + q_{3\nu_3}q_{3\sigma_3}Q^{a_1a_2a_3a_4}_{\nu_1\nu_2\sigma_3\nu_4} + q_{4\nu_4}q_{4\sigma_4}Q^{a_1a_2a_3a_4}_{\nu_1\nu_2\nu_3\sigma_4}] + \frac{43C}{24}\left[\frac{q_{2\nu_2}q_{2\sigma_2}}{q_2^2}Q^{a_1a_2a_3a_4}_{\nu_1\sigma_2\nu_3\nu_4} + \frac{q_{3\nu_3}q_{3\sigma_3}}{q_3^2}Q^{a_1a_2a_3a_4}_{\nu_1\nu_2\sigma_3\nu_4} + \frac{q_{4\nu_4}q_{4\sigma_4}}{q_4^2}Q^{a_1a_2a_3a_4}_{\nu_1\nu_2\nu_3\sigma_4}\right]$
Counterterm vertex diagram: Figure 11	$(Z_g^2\,/\,Z_A-1)\,Q^{a_1a_2a_3a_4}_{\nu_1\nu_2\nu_3\nu_4}$
Counterterm tree-structure diagram: Figure 12 + (q_2, q_3, q_4) permutations	$4(1-Z_A)Q^{a_1a_2a_3a_4}_{\nu_1\nu_2\nu_3\nu_4} - \frac{Z_A(Z_\alpha-1)}{q_1^2+q_2^2+q_3^2+q_4^2} \times [q_{1\nu_1}q_{1\sigma_1}Q^{a_1a_2a_3a_4}_{\sigma_1\nu_2\nu_3\nu_4} + q_{2\nu_2}q_{2\sigma_2}Q^{a_1a_2a_3a_4}_{\nu_1\sigma_2\nu_3\nu_4} + q_{3\nu_3}q_{3\sigma_3}Q^{a_1a_2a_3a_4}_{\nu_1\nu_2\sigma_3\nu_4} + q_{4\omega_4}q_{4\sigma_4}Q^{a_1a_2a_3a_4}_{\nu_1\nu_2\nu_3\sigma_4}] + Z_A(Z_\alpha-1) \times \left[\frac{q_{2\nu_2}q_{2\sigma_2}}{q_2^2}Q^{a_1a_2a_3a_4}_{\nu_1\sigma_2\nu_3\nu_4} + \frac{q_{3\nu_3}q_{3\sigma_3}}{q_2^2}Q^{a_1a_2a_3a_4}_{\nu_1\nu_2\sigma_3\nu_4} + \frac{q_{4\nu_4}q_{4\sigma_4}}{q_4^2}Q^{a_1a_2a_3a_4}_{\nu_1\nu_2\nu_3\sigma_4}\right]$

[a]The $\ln l^2\mu^2$ terms are in units of $[q_2^2q_3^2q_4^2(q_1^2+q_2^2+q_3^2+q_4^2)]^{-1}$; here $C = Ng^2/(16\pi^2)\ln l^2\mu^2$

to do permutations between them for the diagrams of Figures 9, 10, and 12. The final contribution of the four-point renormalized and corresponding counterterm diagrams of Figures 9–12 must be equal to zero. Inserting the value of Z_A defined in (20) under such conditions and equating to zero the coefficients of each tensor separately, one can calculate $Z_g^{(4)}$ and $Z_\alpha^{(4)}$:

$$\begin{aligned} Z_\alpha^{(4)} &= 1+\frac{43}{24}\frac{Ng^2}{16\pi^2}\ln l^2\mu^2 \\ Z_g^{(4)} &= 1-\frac{13}{6}\frac{g^2N}{16\pi^2}\ln l^2\mu^2 \end{aligned} \tag{37}$$

From (37) and (31) one can see that the constant g and the parameter of gauge fixing α satisfy the Ward identity. On the other hand, the Zwanziger gauge-fixing term does not break gauge invariance. Here the β function for g can be defined in the standard way (see Ramond, 1981):

$$\beta=\frac{\partial g}{\partial \ln \mu}=-\frac{11}{3}\frac{g^3N}{16\pi^2}$$

which corresponds to value of the Feynman gauge.

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